Chapter 3
Section 3.4-3.5

Exercise: Graphs of Polynomial Functions

Draw a good graph of the following functions which demonstrates what happens at all x-intercepts.

a) $a(x) = x^3 - 3x + 2$

b) $b(x) = -3(3x-4)^2(2x+1)^4$

c) $c(x) = (x+2)^2(x-5)^3$

d) $d(x) = -x^4 + 4x^3 - 4x^2$

e) $e(x) = -x^3 + 4x$

f) $f(x) = x^3 + 4x^2 - x - 4$

Exercise: Inequalities of Polynomial Functions

Use the graphical method or the test-point method to solve the following polynomial inequalities.

a) $a(x) \leq 0$

b) $b(x) > 0$

c) $x^3 - 3x^2 - 4x + 12 > 0$

d) $d(x) \geq 0$

e) $x^4 + 4 > 5x^2$

f) $2x^2 - x^4 \leq 0$

g) $x^3 + x^2 + 2x - 4 > 0$

Exercise: Asymptotes of Rational Functions

Find all horizontal, vertical and slant asymptotes for the following rational functions.

a) $g(x) = \frac{x^2-4}{x+1}$

b) $h(x) = \frac{1}{x^2-4}$

c) $i(x) = \frac{3x-1}{x+1}$

d) $j(x) = \frac{x^2}{x+1}$

e) $k(x) = \frac{x^2-3}{x}$

Exam 2
Test Review

Topics Covered

1. Definition of Functions
2. Invertible Functions
3. Piecewise Functions
4. Domain and Range of Functions
5. Operations of Functions
6. Transformation Families of Functions
7. Functions of Variation
8. Vertex and Standard Form of Quadratic Functions
9. Finding Roots of Polynomial and Rational Functions
10. Solving Polynomial and Rational Inequalities

Most Commonly Missed Quiz Questions

1. Quiz 4 #2
2. Quiz 4 #3
3. Quiz 6 #1

Strategies for Factoring Polynomials

1. Find the Greatest Common Factor and factor it out.
2. Factoring by Grouping.
3. Factoring equations of Quadratic Type.
4. Using Quadratic Formula on quadratic polynomials.
5. Find Possible Rational Zeroes and test them using synthetic division.

Graphs of Polynomial Functions

$a(x) = x^3 - 3x + 2$ Possible $\mathbb{Q}$ (rational) zeroes: $\pm 1, \pm 2$

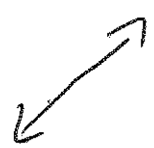
$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

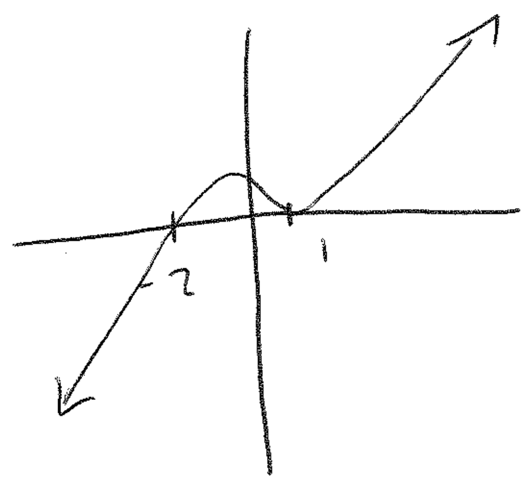
$$\begin{aligned} a(x) &= (x-1)(x^2+x-2) \\ &= (x-1)(x+2)(x-1) \\ &= (x-1)^2(x+2) \end{aligned}$$

1 root of mult. 2

-2 root of mult. 1.

End Behaviors:



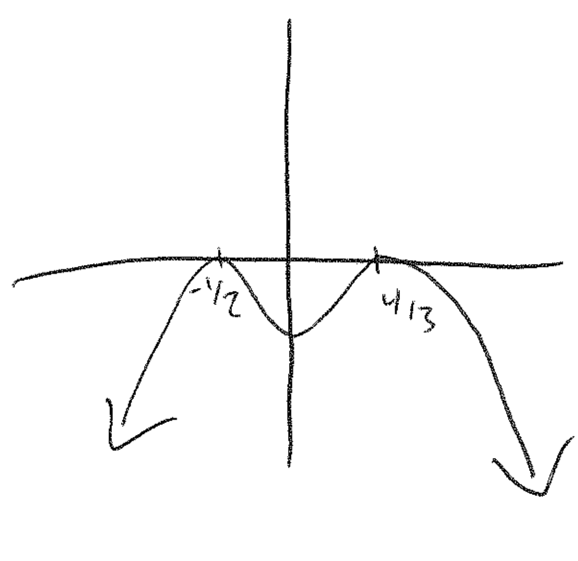


$b(x) = -3(3x-4)^2(2x+1)^4$

$\frac{4}{3}$ root of mult. 2

$-\frac{1}{2}$ root of mult. 4

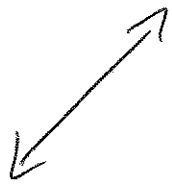
End Behaviors

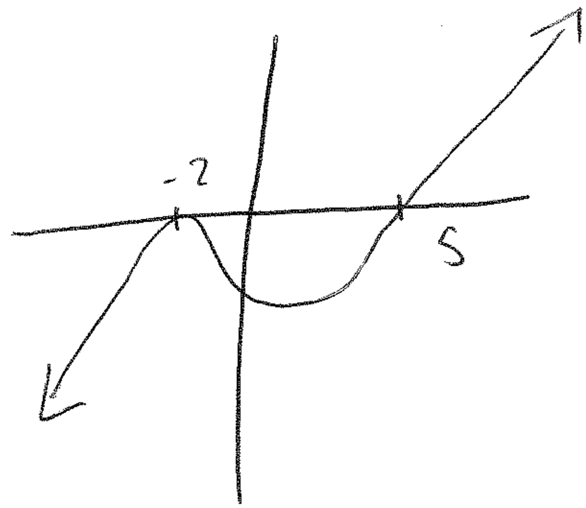


$e(x) = (x+2)^2(x-5)^3$

-2 root of mult. 2
5 root of mult. 3

End Behaviors

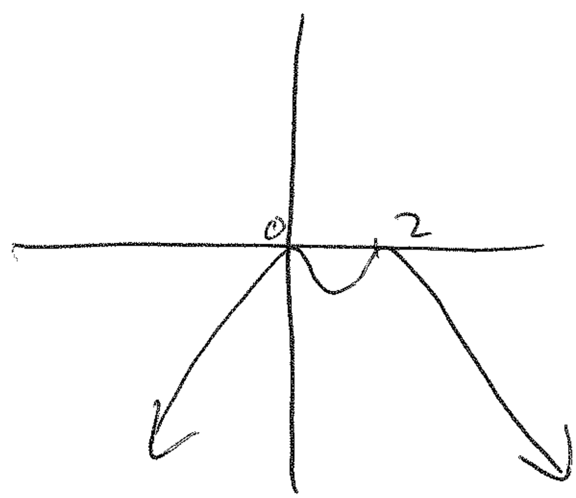




$d(x) = -x^4 + 4x^3 - 4x^2$
$= -x^2(x^2 - 4x + 4)$
$= -x^2(x-2)^2$

2 root of mult. 2
0 root of mult. 2

End Behaviors



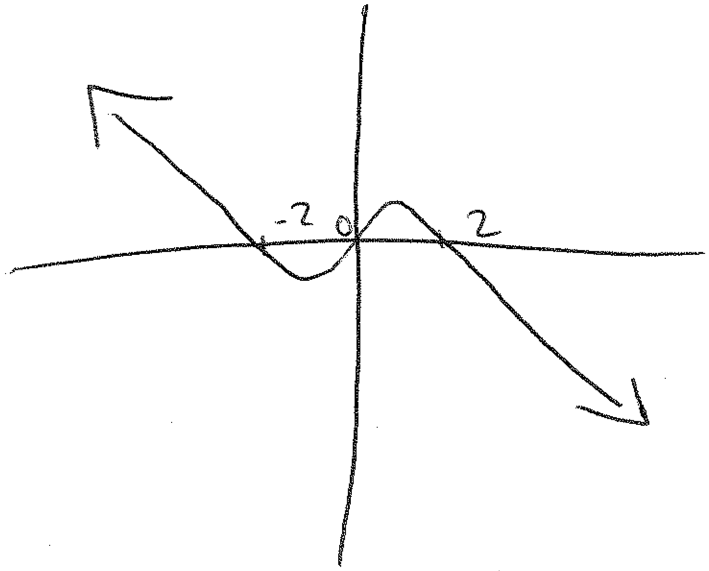
$e(x) = -x^3 + 4x$

$= -x(x^2 - 4)$

$= -x(x+2)(x-2)$

0, 2, -2 all roots of mult. 1

<u>End Behaviors</u>



$f(x) = x^3 + 4x^2 - x - 4$

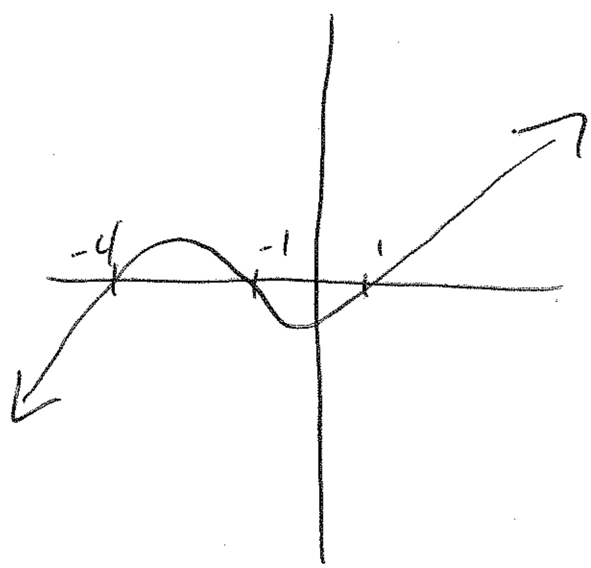
$= x^2(x+4) - (x+4)$

$= (x^2 - 1)(x+4)$

$= (x-1)(x+1)(x+4)$

1, -1, -4 all roots of mult. 1.

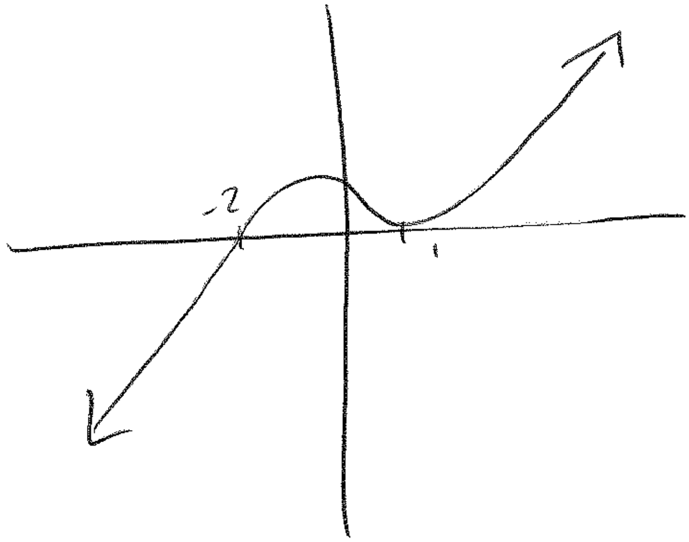
<u>End Behaviors</u>



Inequalities of Polynomial Functions

$a(x) \le 0$

$x^3 - 3x + 2 \le 0$



$x^3 - 3x + 2 = 0$
at $-2, 1$

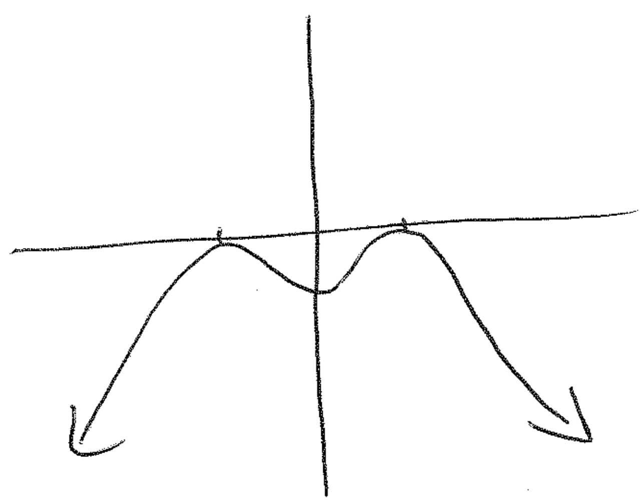
Solution $(-\infty, -2] \cup [1, 1]$

or more properly

$(-\infty, -2] \cup \{1\}$.

$b(x) > 0$

$-3(3x-4)^2(2x+1)^4 > 0$



Solution

ϕ or nowhere

$x^3-3x^2-4x+12>0$

First find roots

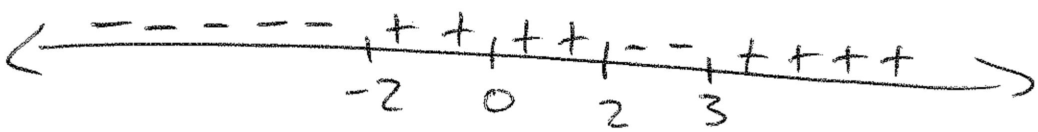
Possible $\mathbb{Q}$ zeroes: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

1, -1 don't work

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$x^3-3x^2-4x+12=(x-2)(x^2-x-6)$$
$$=(x-2)(x-3)(x+2)$$

Roots at 2, 3, -2 all of mult. 1



Test Points:

$x=-3$ $\quad (-3-2)(-3-3)(-3+2)=$ 3 negatives = negative

$x=0$ $\quad (0-2)(0-3)(0+2)=$ 2 negatives = positive

$x=2.5$ $\quad (2.5-2)(2.5-3)(2.5+2)=$ 1 negative = negative

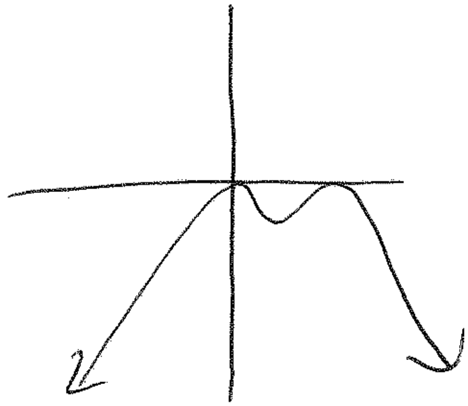
$x=4$ $\quad (4-2)(4-3)(4+2)=$ 0 negative = positive

Solution

$$(-2,2)\cup(3,\infty)$$

$d(x) \geq 0$

$d(x) = 0$
at $0, 2$



<u>Solution</u>

$\{0, 2\}$

$x^4 + 4 > 5x^2$ Quadratic type, let $x^2 = u$

$x^4 - 5x^2 + 4 > 0$

$u^2 - 5u + 4 > 0$

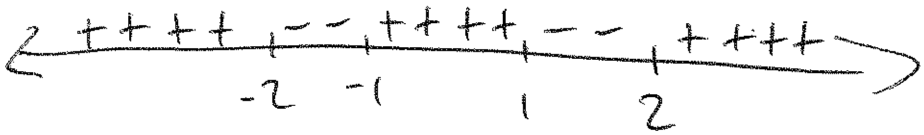
$(u-4)(u-1) > 0$

~~Roots at 4,1 of mult 1~~

$(x^2-4)(x^2-1) > 0$

$(x-2)(x+2)(x-1)(x+1) > 0$

<u>Test Points</u>



<u>Solution</u>

$(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$

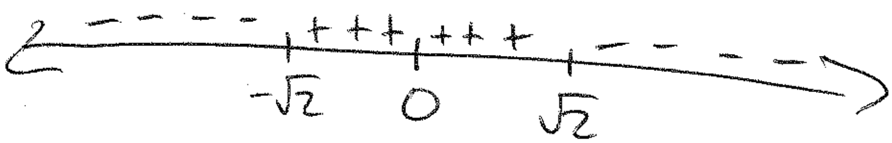
$2x^2 - x^4 \le 0$

$-x^2(x^2-2) \le 0$

$-x^2(x-\sqrt{2})(x+\sqrt{2}) \le 0$

Roots at $\pm\sqrt{2}$ of mult. 1
Root at 0 of mult. 2

Test Points:



Solution

$(-\infty, \sqrt{2}] \cup [\sqrt{2}, \infty)$.

$x^3 + x^2 + 2x - 4 > 0$

Possible $\mathbb{Q}$ zeroes: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 2 & -4 \\ & & 1 & 2 & 4 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

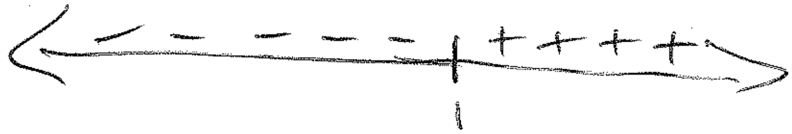
$x^3 + x^2 + 2x - 4 = (x-1)(x^2+2x+4)$

Quadratic Formula: $\dfrac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 4}}{2}$

Discriminant is negative so no real roots.

Only real root is 1

Test Points:



Solution: $(1, \infty)$

Asymptotes of Rational Functions

$g(x) = \dfrac{x^2-4}{x+1}$ $\quad x+1=0$ at -1 so ver. asymp. at $x=-1$.

degree of x^2-4 is 1 greater than degree of $x+1$

so divide

$$\begin{array}{r|rrr} -1 & 1 & 0 & -4 \\ & & -1 & 1 \\ \hline & 1 & -1 & -3 \end{array}$$

$g(x) = (x-1) + \dfrac{-3}{x+1}$. So slant asymp. at $y = x-1$.

$h(x) = \dfrac{1}{x^2-4}$ $\quad (x^2-4) = (x+2)(x-2)$ so vert. asymp. at $x=2$ and $x=-2$.

degree of x^2-4 is greater than degree of 1,

so hor. asymp. at $y=0$.

$i(x) = \dfrac{3x-1}{x+1}$ $\quad$ ver. asymp. at $x=-1$.

degree of $3x-1$ equals degree of x so

hor. asymp. at $y = \frac{3}{1} = 3$.

$j(x) = \dfrac{x^2}{x+1}$ ver. asymp. at $x = -1$

$$\begin{array}{r|rrr} -1 & 1 & 0 & 0 \\ & & -1 & 1 \\ \hline & 1 & -1 & 1 \end{array}$$

$j(x) = (x-1) + \dfrac{1}{x+1}$ so

slant asymp. at $y = (x-1)$.

$k(x) = \dfrac{x^2-3}{x}$ ver. asymp. at $x = 0$

$$\begin{array}{r|rrr} 0 & 1 & 0 & -3 \\ & & 0 & 0 \\ \hline & 1 & 0 & -3 \end{array}$$

$k(x) = x + \dfrac{-3}{x}$ so

slant asymp. at $y = x$.